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Quadrature-difference methods for solving linear and nonlinear singular integro-differential equations

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ABSTRACT

Here we propose and justify quadrature-difference methods for solving different kinds (linear, nonlinear and multidimensional) of periodic singular integro-differential equations.

1. Introduction

In Section 2 we propose and justify quadrature-difference methods for solving linear and nonlinear singular integro-differential equations with Hölder continuous coefficients and right-hand sides. In Section 3 the same methods are justified for linear singular integro-differential equations with discontinuous coefficients and right-hand sides. In Section 4 we propose and justify cubature-difference methods for solving multidimensional singular integro-differential equations in Sobolev space.

2. Linear and nonlinear singular integro-differential equations with continuous coefficients

Let us consider linear

$$\sum_{\nu=0}^{m}(a_\nu(t)x^{(\nu)}(t)+b_\nu(t)(Jx^{(\nu)})(t)+(J^0h_\nu x^{(\nu)})(t))=y(t) \tag{2.1}$$

and nonlinear

$$F(t,x^{(m)}(t),\dots,x(t),(Jx^{(m)})(t),\dots,(Jx)(t),(J^0h_m x^{(m)})(t),\dots,(J^0h_0x)(t))=y(t) \tag{2.2}$$

singular integro-differential equations where $x(t)$ is the desired unknown, $a_\nu(t), b_\nu(t), h_\nu(t,\tau)$, $\nu=0,1,\dots,m-1$, $y(t)$ and $F(t,u_m,\dots,u_0,v_m,\dots,v_0,w_m,\dots,w_0)$ are given continuous functions, 2π-periodic in the variables t,τ, singular integrals

$$(Jx^{(\nu)})(t)=\frac{1}{2\pi}\int_0^{2\pi}x^{(\nu)}(\tau)\cot\frac{\tau-t}{2}\mathrm{d}\tau,\quad \nu=0,1,\dots,m-1,$$

are to be interpreted as the Cauchy–Lebesgues principal values and

$$(J^0h_\nu x^{(\nu)})(t)=\frac{1}{2\pi}\int_0^{2\pi}h_\nu(t,\tau)x^{(\nu)}(\tau)\mathrm{d}\tau,\quad \nu=0,1,\dots,m-1,$$

are regular integrals.

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